ACCELERATION OF A PULSATING PLASMA SLUG

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This paper deals with the acceleration of a pulsating plasma slug in a high current pulse accelerator. The pulses of the plasma slug are caused by the magnetic pressure forces of interior currents and the gas-kinetic pressure forces in the slug. With an increase in gas-kinetic pressure these pulses decrease and can be absent altogether. In this case, only an expansion of the slug occurs. Strong pulses, sharply changing the system's inductance, can cause an overloading of the system and the appearance of instability of the pinch type. Also, the movement of the slug's center of mass due to the force of magnetic pressure is examined.

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1. A number of theoretical and experimental papers have studied electrodynamic acceleration of plasma slugs [1--7]. This is connected with the effectiveness of accelerating quasineutral plasma slugs to speeds of the order of 10^6 m [meters] /second at densities of $10^{14} \div 10^{20}$ particles/centimeter^3 in high-current plasma accelerators. These accelerators are used extensively in injecting plasma into thermonuclear devices, for creating strong shock waves and other purposes.

Usually a slug is thought of as an undistorted conducting bridge between electrodes, accelerated by magnetic pressure forces of internal currents passing in the accelerator through the bridge. The basis of such a conception is that the plasma slug is distinctly localized in width and is a stable unit in the acceleration process.

However, a slug is a very mobile and unstable formation obtained by the action of electrodynamic and gas dynamic forces. The initial stage of slug formation can be followed using a model of formation of a spark channel by capacitor discharge on electrodes. Works [8--14] give a detailed analysis of this stage. The general process of channel formation occurs in the following way.

With sudden application of a voltage across the gas discharge gap between electrodes, shock ionization of gas is caused by electronic Townsend ionization or streaming, depending upon the initial pressure in the gas. A thin channel that readily conducts a current is formed. As a result of the discharge of a capacitor with a large capacitance of electric energy

$$W_{эл} = \frac{C_o U_o^2}{2}$$

a great amount of energy is evolved in this channel in the form of a Joule effect of 1 Joule in a time of the order of 10^{-7} seconds per unit length of the channel. As a result of intensive energy evolution and temperature and pressure inside the channel sharply increase, and the channel begins to widen, modeling itself as a cylindrical source. Dispersion of such a channel will continue until the action of the energy evolved in the channel is compensated by its drawing off into the surrounding medium by thermal exchange or other means.

In particular, at great current strengths of the order of $6 \cdot 10^5$ amps, present in a pulse accelerator, magnetic pressure forces of internal currents are of extremely great importance, causing electrodynamic acceleration of slugs to high speeds and, on the other hand, impeding gas dynamic dispersion due to the pinch effect.

The pinch phenomenon in pure form has been studied in works [15, 16]. Pinch pulses result in instability of the pinch, and in the final stage, in its collapse. Work [5] attempted to examine the influence of pulses on the acceleration process; however, the complexity of the equations did not permit analytical solution of this problem.

2. Let us examine the acceleration process of a pulsating slug in a railtron [rel'sotron] or a coaxial. Fig. 1 gives the equivalent electromechanical circuit of such an accelerator.

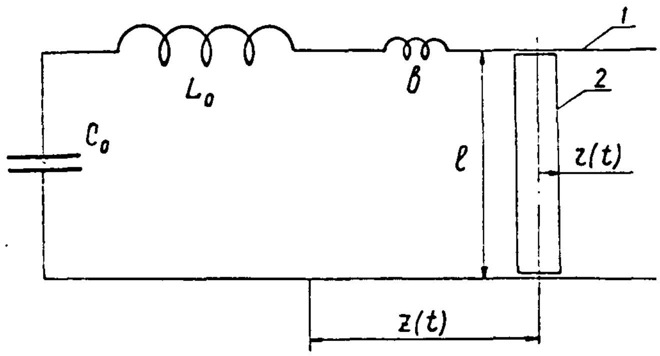

Fig. 1. Equivalent Electric Circuit of a Pulse Plasma Accelerator

1 -- lead electrodes; 2 -- plasma slug; C_0 -- capacitor capacitance; L_0 -- initial inductance; b -- distributed inductance of a unit length of the accelerator; l -- distance between electrodes; z(t) -- coordinate of slug's center of mass; r(t) -- coordinate of slug surface.

Here the slug is given in the form of a mobile linear pinch 2 of radius r(t), accelerated along the lead electrodes 1 by magnetic pressure forces of currents passing through the circuit with capacitor discharge C_0. The inductance of the circuit will accumulate from the initial circuit inductance L_0, used as the inductance of the accelerator, which increases with acceleration of the slug proportional to the coordinate of its center of gravity, $L_1 = bz(t)$, where b is the inductance distributed on a unit length of the accelerator itself, and the inductance of the pulsating pinch

$$L_2 = \frac{l\mu}{2\pi} \ln \frac{r_0}{r(t)},$$

where l is the distance between electrodes and r(t) is the pinch radius at any moment of time. In this way,

$$L = L_0 + bz(t) + \frac{l\mu}{2\pi} \ln \frac{r_0}{r(t)}. \tag{1}$$

We write the equation describing the pulsation of a plasma slug, following work [5], in the form

$$\frac{M}{4} \frac{d^2 r}{dt^2} = -\frac{l\mu}{2\pi} \frac{I^2}{2r} + 2\pi l r_0 p_0 \left(\frac{r_0}{r}\right)^{7/3}, \tag{2}$$

where M -- mass of accelerated slug
μ -- dimension coefficient equalling $4_\pi \cdot 10^{-7}$ h [henries]/m;
$\frac{1}{4}$ -- power taking into account an equal distribution of mass by slug cross-section;
p_0 -- initial gas pressure in pinch;
r_0 -- initial pinch radius.

Here it is proposed that the forces of inertia (left member) are balanced by the forces of magnetic compression (first term of right member) and the gas-dynamic forces, whence gas compression occurs adiabatically with indicator γ = 5/3.

Discharge current I is determined by the equation of discharge continuity

$$I = -C_0 \frac{dU}{dt}. \tag{3}$$

For completeness we add the equation of the electric circuit

$$\frac{d}{dt} LI + RI + \frac{1}{C_0} \int_0^t I\, dt = 0 \tag{4}$$

and the equation of motion of the plasma slug's center of mass

$$M \frac{d^2 z}{dt^2} = \frac{b}{2} I^2. \tag{5}$$

We give the initial conditions in the form

at t = 0

$$z=0; \quad \frac{dz}{dt}=0;$$
$$r=r_0; \quad \frac{dr}{dt}=0; \qquad (6)$$
$$U=U_0; \quad I=0.$$

Equation system (1) -- (5) with initial conditions (6) will describe a simple model of pulsating slug acceleration in full. This system is essentially nonlinear and can be solved on computers. For this it is convenient to convert to dimensionless magnitudes. Introducing dimensionless variables

$$y=\frac{b}{L_0}z; \quad x=\frac{r}{r_0}; \quad \varphi=\frac{U}{U_0}; \quad \tau=\omega t, \qquad (7)$$

we write equation system (1) -- (5) in the form

$$\frac{d^2y}{d\tau^2}=q\left(\frac{d\varphi}{d\tau}\right)^2, \qquad (8)$$

$$\frac{d^2x}{d\tau^2}=-qp\frac{1}{x}\left(\frac{d\varphi}{d\tau}\right)^2+\nu x^{-7/3}, \qquad (9)$$

$$\frac{d}{d\tau}\left[\left(1+y+\varepsilon\ln\frac{1}{x}\right)\frac{d\varphi}{d\tau}\right]+\alpha\frac{d\varphi}{d\tau}+\varphi=0 \qquad (10)$$

with dimensionless parameters

$$q=\frac{C_0^2U_0^2b^2}{2ML_0}; \quad p=\frac{4\mu l L_0}{2\pi r_0^2 b^2}; \qquad (11)$$

where

$$\alpha=R\sqrt{\frac{C_0}{L_0}}; \quad \nu=\frac{8\pi p_0 l}{M\omega^2}; \quad \varepsilon=\frac{\mu l}{2\pi L_0},$$

$$\omega=\frac{1}{\sqrt{L_0C_0}}.$$

c)

The initial conditions with the new variables will be: at $\tau = 0$

$$y=0; \quad \frac{dy}{d\tau}=0;$$

$$x=1; \quad \frac{dx}{d\tau}=0; \qquad (12)$$

$$\varphi=1; \quad \frac{d\varphi}{d\tau}=0.$$

3. Let us appraise the value of magnitudes of the parameters entering into equations (8) -- (10) and their effect on the system's behavior.

The magnitude of parameter q in actual accelerators can change in the wide range $q = 10^{-3} \div 10^{2}$.

This influence on the acceleration process was studied in work [1]. In order to allow the possibility of comparing the calculation results with those of earlier works, the value of this parameter is taken in the interval $0.1 \div 10$. This is also justifiable because the optimal values of this parameter lie in this interval, as the calculations show.

The magnitude of parameter α is determined by ohmic losses in the accelerator and was taken to equal 0.1 in accordance with [7] and corresponding to the value in real accelerators [2]. Parameter p is determined by the geometric characteristics of the accelerator and can assume various values in a wide range. If $l < 0.01$ m, then b^2 can be large; at $l \sim 0.1 \div 0.01$ m, $b = (1 \div 4) \cdot 10^{-7}$ h/m; L_0 is determined by the initial eddy inductance, and its magnitude is now reduced to $(100 \div 50) \cdot 10^{-9}$ h; $r_0 \sim 0.01 \div 0.001$ m. In this problem, the magnitude of parameter p was taken equal to $1 \div 100$, whence lower values correspond to small l and large b.

The magnitude of parameter ε changes only with a change in accelerator structure and is taken to equal 0.1, which at $L_0 = 50 \cdot 10^{-9}$ h corresponds to the magnitude $l = 0.025$ m.

The magnitude of parameter v is determined by the relationship of gas pressure forces within the column to the forces of inertia. An increase in v signifies a physical increase in the initial pressure within the slug. It may occur that this pressure is great in comparison with the magnetic pressure. Then the slug does not pulsate, and gas-dynamic expansion of the slug will occur.

The behavior of solutions of the nonlinear system will be determined by the relationship of parameter magnitudes. Analytical solution of this system can be carried out by asymptotic methods only in the case of small values of parameters q, α, v, when the equation can be considered as an equation with little nonlinearity.

4. Numerical solution of equation system (8) -- (10) with initial conditions (12) by the Range-Kutt method was carried out on a "Ural" computer.

Figs. 2, 3, and 4 give the results of these equations for parameter values $q = 0.1$; $\alpha = 0.1$; $\varepsilon = 0.1$; $v = 0.01 \div 0.1$; $p = 1$.

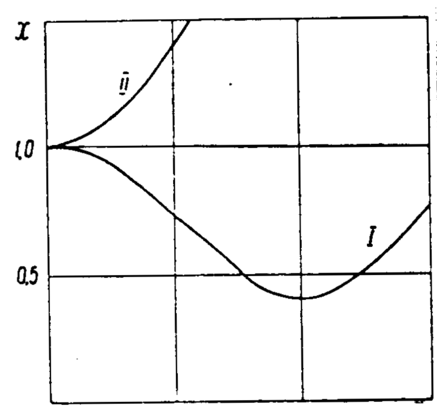

Fig. 2. Change of Slug Radius x in Time τ at Different Accelerator Parameters:

I -- $q = 0.1$; $\alpha = 0.1$; $\varepsilon = 0.1$; $v = 0.01$; $p = 1$;
II -- $q = 0.1$; $\alpha = 0.1$; $\varepsilon = 0.1$; $v = 0.1$; $p = 1$.

Analyzing the graphs, we see that with acceleration of the pulsating plasma slug two modes of acceleration are possible: 1) mode with an expanding plasma slug (curve II on Fig. 2); 2) mode with a pulsating plasma slug (curve I on Fig. 2). For convenience in comparison, these curves were obtained with identical magnitudes of parameters entering into equations (8) -- (10): $q = 0.1$; $\alpha = 0.1$; $\varepsilon = 0.1$;

p = 1. Only the magnitude of parameter v changed, proportionally to the gas kinetic pressure within the slug.

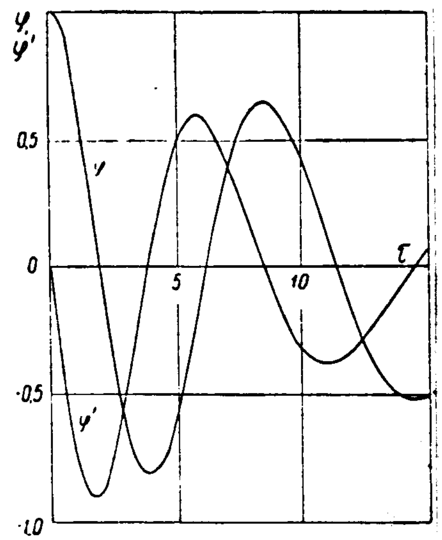

Fig. 3. Change in Voltage φ and Current φ' in time τ with the Following Accelerator Parameters:

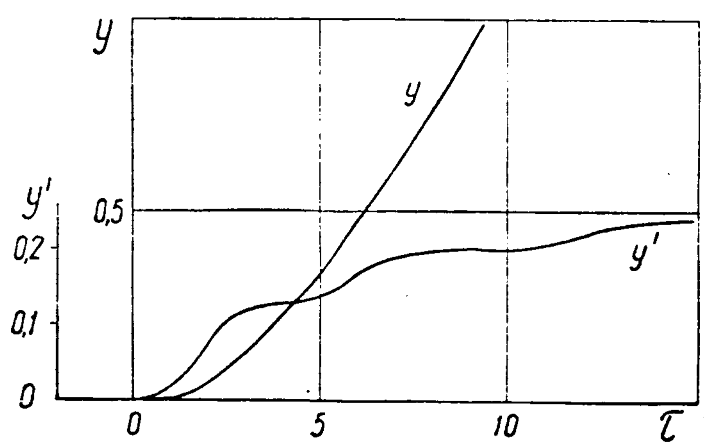

Fig. 4. Change in Speed of Slug's Center of Mass y' and Path y running through the slug at the following accelerator parameters:

As can be seen from these graphs, at $v = 0.01$ the slug has a pulsating motion. The slug compresses by 2.5 times and then expands without limit -- pinch type instability appears. Increase of parameter v by 10 times at fixed values of the remaining parameters results in an expansion of the plasma slug with respect to its center of mass. In both cases an acceleration of the slug's center of mass occurs along the lead electrodes by the action of magnetic pressure forces.

On the basis of numerous calculations we can form general criteria for establishing the mode with a pulsating or an expanding plasma slug; at $q \cdot p > 10v$ pulsation will occur and at $q \cdot p \sim v$ the slug will expand.

Physically these conditions signify that in the first case the forces of magnetic compression in the initial moment exceed the forces of gas kinetic pressure within the slug; in the second case these forces are comparable in magnitude, However, the action of magnetic forces also causes the movement of the slug as a whole.

Fig. 3 gives the change of the dimensionless current φ' and voltage φ in time. Fig. 4 illustrates the change in speed of movement of the slug's center of mass y' and its path y in time.

It appears that in the examined case of small parameter values $q \ll 1$; $p \sim 1$; $v \ll 1$ the mode with pulsation or expansion of the slug influences the accelerator processes in an identical degree. This can be explained by the fact that at small magnitudes of energy parameter q a slow change in the slug dimensions and its inductance occurs. Comparing these graphs with those of [1, 7], we see that the character of slug acceleration as a whole remains as before. On the strength of small values of energy parameter q, a change in current and voltage occurs as in a system with little nonlinearity and decay. The magnitude of half-life somewhat increased at the expense of increase in the circuit's self-inductance.

Increase in parameter q greatly affects the behavior of the solutions, causing strong slug pulsations and its intense acceleration. The slug radius sharply decreases with an increase in q, causing a significant increase in the slug's inductance and a significant increase of inductive voltage drop along the slug, which results in the appearance of a voltage along the slug and more intense development of instability. In connection with this, the mode with a pulsating slug is less effective from the point of view of accelerating the slug as a whole in comparison with the mode of acceleration with expansion of the slug, although in the latter case part of the energy is wasted on motion in the reverse direction.

Inasmuch as acceleration of the slug's center of mass occurs at the expense of magnetic pressure forces, the second criterion of the theory is that the magnetic pressure forces considerably exceed the gas pressure forces in the coaxial. With the applied voltages of magnetic fields $\sim$ 1 TL [Тл], this condition is already fulfilled at pressures of the order 10^3 newtons/m^2.

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